Cartesian and 2-additive opindexed categories

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Abstract. [1]
We first give a characterization of cartesian objects in the cartesian 2-category OplCat of opindexed categories. It turns out that any such object is given by a pseudofunctor $\Phi\colon \mathbb{B} \to \mathsf{Cat}$, where $\mathbb{B}$ has finite products and, considering the consequent canonical oplax monoidal structure $\mathcal{L}$ on Φ, $\mathcal{L}$ admits a right adjoint $\mathcal{R}$, which makes Φ a lax monoidal pseudofunctor. As a consequence, we find that *discrete* cartesian objects are nothing but finite-product preserving functors $\Phi\colon \mathbb{B} \to \mathsf{Set}$. When moreover $\mathbb{B}$ is additive, this means that Φ factorizes through the category Ab of abelian groups, and such discrete cartesian opindexed categories can be equivalently described as additive functors into Ab.

As a further step, we consider the intermediate case of opindexed groupoids (corresponding, via the Grothendieck construction, to opfibrations with groupoidal fibers). It turns out that cartesian opindexed groupoids correspond to pseudofunctors preserving finite products up to equivalences. Again, considering the special case where moreover B is additive, we find that any such $\Phi\colon \mathbb{B} \to \mathsf{Gpd}$ factorizes through the 2-category $\mathbb{S}\mathsf{ym2}\mathbb{G}\mathsf{p}$ of symmetric 2-groups. As a final result, we characterize the latter as 2-additive pseudofunctors (in the sense of Dupont [?]), from which the name of 2-additive opindexed categories.

References

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[1]Work in progress, jointly with A. S. Cigoli and G. Metere